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Robust Airfoil Shape Optimization Using Design for Six Sigma

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Nomenclature

C_D	=	drag coefficient
C_L	=	lift coefficient
C_L^*	=	target lift coefficient
M	=	freestream Mach number
M_L, M_U	=	freestream Mach numbers at lower and upper bounds
$\bar{M}$	=	set of randomly selected freestream Mach numbers
α	=	angle of attack
μ	=	standard deviation
σ	=	mean value

Introduction

BECAUSE of the rapid development of computer technologies, tremendous achievement has been made in the computational fluid dynamics (CFD) area during the past few decades. Currently, CFD techniques are widely used not only to simulate and understand complicated flow physics, but also to optimize the aerodynamic shape of air vehicles that enables maximum performance at desired operating conditions. One of the most popular aerodynamic shape optimization algorithms currently available is the gradient-based optimization technique in which a specified objective function is minimized. Sensitivities, the gradients of the objective function with respect to design variables, are used to update the design variables in the direction that the magnitude of the objective function can be reduced in a systematic way.

However, application of the aerodynamic shape optimization has been mostly made to single-point optimizations[1–4] in which the optimal shape is determined for a specific flight condition. Even though the single-point design optimization guarantees good performance at the specified design point, practical experience indicates that it can lead to a dramatically inferior performance even at a slightly off-design operating condition caused by local gusts or atmospheric disturbances frequently encountered in actual flights. This problem may also occur due to errors involved in manufacturing, which cannot be completely avoided in a practical sense.

A straightforward method of avoiding this difficulty is to consider multiple flight conditions simultaneously by constructing the objective function as a weighted average of the flight conditions.[5–9] Even though improved performance can be achieved over a wide range of flight conditions by using this multipoint optimization method, practical problems still exist in the selection of the desired flight conditions and their weighting factors. Currently, a definite theoretical principle of the selection does not exist, and it is strongly dependent on the designer's experience and intuition. Also, multipoint design optimization does not necessarily guarantee good performance at flight conditions off from the given discrete design points.

To relieve the shortcoming of the multipoint design optimization and to achieve a consistent performance at varying operating conditions, a robustness can be adopted in the design process. Recently, an intrinsically probabilistic approach[10–14] has been incorporated in the aerodynamic shape optimization. In this robust design approach, the flight Mach number was treated as a probabilistic variable, and the integrated sum of the aerodynamic drag over a prescribed Mach number range was set as the objective function. When this was done, the overall drag for the given range of flight Mach number was reduced when the objective function was minimized. However, the results revealed that the drag divergence was not effectively eliminated from the desired flight Mach number range because of the negligence of the minimal variance of the performance.

In the present study, a robust airfoil shape optimization technique has been developed by adopting the concept of design for six sigma,[15] which has been commonly used in quality engineering to minimize the variance of products. The uncertain changes in the flight condition were implemented in the numerical optimization by incorporating a probabilistic approach based on the stratified random sampling of the flight Mach number.[14] Both mean and standard deviation of the drag coefficient were used in constructing the objective function, so that the overall drag and the variance of the performance can be minimized simultaneously, regardless of the changes in the flight condition. When this was done, in addition to overall drag minimization, the drag divergence was effectively controlled over the flight Mach number range of interest. The present method was applied to the aerodynamic shape optimization of two-dimensional airfoils in transonic flow regimes. The results were compared with those obtained by other optimization methods for validation.

Mathematical Modeling

Numerical Method

The governing Euler equations of the flow solver were discretized using a cell-centered finite volume method on unstructured meshes. The inviscid flux across each cell face was computed based on Roe's flux-difference splitting. To obtain second-order spatial accuracy, estimation of the flow variables at each cell face was achieved by interpolating the solution using a Taylor series expansion in the neighborhood of each cell center. The cell-averaged solution gradient required at the cell center for the preceding expansion was computed from the Gauss theorem. The expansion also requires nodal values of the solution, which can be computed from the surrounding cell center data using a second-order accurate pseudo-Laplacian averaging. An implicit time-integration algorithm based on the linearized Euler backward differencing was used to advance the solution in time. The linear system of equations was solved at each time step using a point Gauss–Seidel method.

The sensitivities were computed from a continuous adjoint method.[1–4] To obtain accurate sensitivities, the continuous adjoint flux was evaluated by the guidance of the discrete adjoint residual[2–4]

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based on a discrete adjoint formulation. The adjoint equations were solved using numerical methods similar to those for the flow equations, such as the implicit time integration, the high-order reconstruction with pseudo-Laplacian averaging, the Gauss–Seidel time integration, and local time stepping.

The airfoil shape was modified by representing the airfoil section contours using the Hicks–Henne shape functions (see Ref. 16). The minimization of the objective function under the constraint of maintaining the desired lift was achieved using the feasible direction method.[17]

Design for Six Sigma and Robustness

The transonic flow regime is known to be the most fuel efficient, allowing aircraft to operate at high lift-to-drag ratios. However, above a certain critical Mach number, a sudden rise of drag, known as drag divergence, occurs. Thus, the cruising speed of most civil aircraft is restricted below the drag divergence Mach number. If drag divergence can be avoided from the desired range of flight condition, aircraft can operate at higher cruising Mach numbers with better aerodynamic efficiency. However, elimination of the drag divergence has not been considered in previous aerodynamic shape optimization studies.

In the present study, a robust aerodynamic shape optimization technique based on the concept of design for six sigma has been developed to suppress the drag divergence in transonic airfoil design. For this purpose, statistical parameters such as the mean and the variance of the drag coefficient were adopted as a measure of the drag divergence. The objective function may be written as follows: Minimize

$$\sigma^2 + \mu^2 \tag{1a}$$

subject to

$$C_L(M, \alpha) \geq C_L^*, \qquad M \in [M_L, M_U] \tag{1b}$$

where

$$\sigma^2 = \frac{1}{M_U - M_L} \int_{M_L}^{M_U} (C_D(M, \alpha) - \mu)^2 \, \mathrm{d}M \tag{2}$$

$$\mu = \frac{1}{M_U - M_L} \int_{M_L}^{M_U} C_D(M, \alpha) \, \mathrm{d}M \tag{3}$$

Because the integration of drag over a given range of freestream Mach number in Eq. (3) represents the area under the drag profile, μ can be considered as the average drag coefficient. Statistically, the integration can be replaced by the average of an infinite number of population samplings. To incorporate the uncertainty of the operating condition by a finite number of sampling (design) points, a random number generation approach was used to select the design points, $\bar{M} = [M_1, \ldots, M_n] \subset [M_L, M_U]$, where n represents the number of design points. To avoid the possibility of a biased selection of the design points, the stratified sampling method based on equal allocation was used.[14] The robustness of design for the uncertain operating conditions was further enhanced by reselecting the set of $\bar{M}$ during the design process. Frequent reselection of $\bar{M}$ can cause slow convergence, and too sparse application may result in an excessive optimization leading to a localized minimum for a certain set of $\bar{M}$. In the present study, 10 stratified random sampling points were used as the design points, and the reselection of $\bar{M}$ was made at every 3 design iterations, which was equivalent to 1 design cycle. A total of five design cycles were carried out.

Parallel Implementation

For the present robust design, the objective function and its gradients must be evaluated repeatedly for each individual design point selected from the stratified sampling. To expedite the design process, a parallel computation technique was adopted by assigning the evaluation of the objective function and its gradients of each design point to local parallel processors. Thus, the total number of processors was set equal to the number of stratified sampling points such that the robust design can be completed in a turnaround time equivalent to that of single-point designs. The computational overhead for data communication was negligible because only a few data needed to be collected from individual processors. Calculations were made on a Linux-based personal computer-cluster having 2.4-GHz CPUs.

Results and Discussion

To validate the present robust design method, two-dimensional transonic airfoil shape optimization was performed. An NACA0012 airfoil was taken as a baseline configuration, and the airfoil shape was designed to minimize the drag and its variance over a freestream Mach number range between 0.7 and 0.8 under the constraint that the lift coefficient of the designed airfoil is larger than a target lift coefficient of 0.35. The freestream Mach number was chosen as the uncertainty parameter, and 10 design variables were adopted to represent the upper surface of the airfoil. The computational mesh consisted of 6375 triangular cells and 3286 nodes as shown in Fig. 1.

In Fig. 2, comparison of the resultant drag profiles obtained from various optimization methods is presented. It shows that the configuration obtained from single-point design optimization revealed a drastic drag divergence characteristic, even at slightly off-design operating conditions. The result of multipoint design optimization performed for relatively high freestream Mach numbers shows some improvement over that of the single-point optimization by suppressing the rapid increase of drag. However, the overall drag became fairly high as the Mach number increased. The result obtained from a robust design optimization based on overall drag minimization[13] shows a significant reduction in drag for the complete range of the

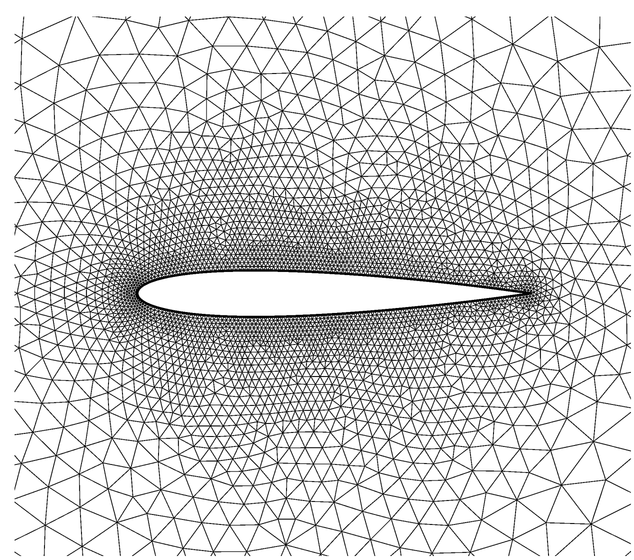

Fig. 1 Partial view of computational mesh.

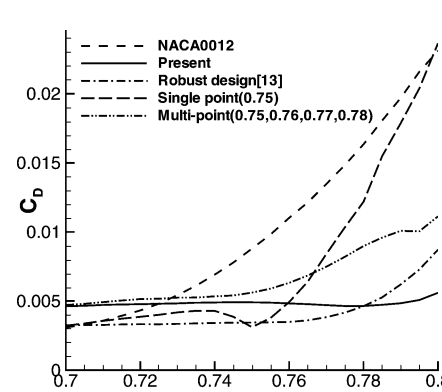

Fig. 2 Comparison of drag profiles before and after design optimization.

flight Mach number. However, because of the lack of the consideration of a consistent performance, this method did not effectively control the rapid increase of drag within the flight range of interest. Meanwhile, the result of the present method shows that, by imposing the drag variance in the objective function, the stiff increase of drag was effectively suppressed throughout the range of the flight Mach number, even though the predicted overall drag was slightly higher at relatively low Mach numbers.

In Fig. 3, comparison of the airfoil surface contours before and after the design optimization is presented. It shows that the leading-edge radius was reduced for all design cases. As a result, the suction peak increases, and this helps to maintain the desired lift. However, the overall profile obtained from the robust design methods shows a significant difference from that of the single-point or multipoint optimization. Even though similar contours were obtained from both robust design methods, the airfoil designed by the present method is slightly thinner at the fore part of the airfoil, inducing less acceleration of the flow.

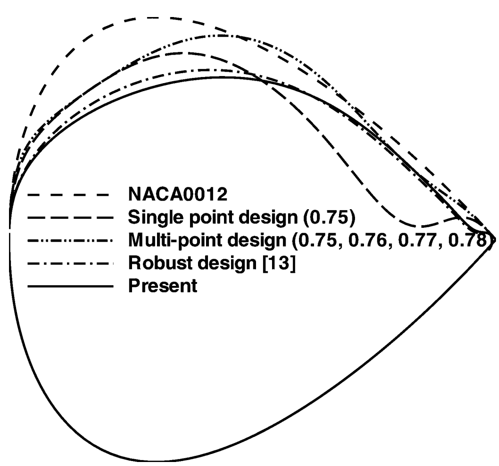

Fig. 3 Comparison of airfoil surface contours before and after design optimization.

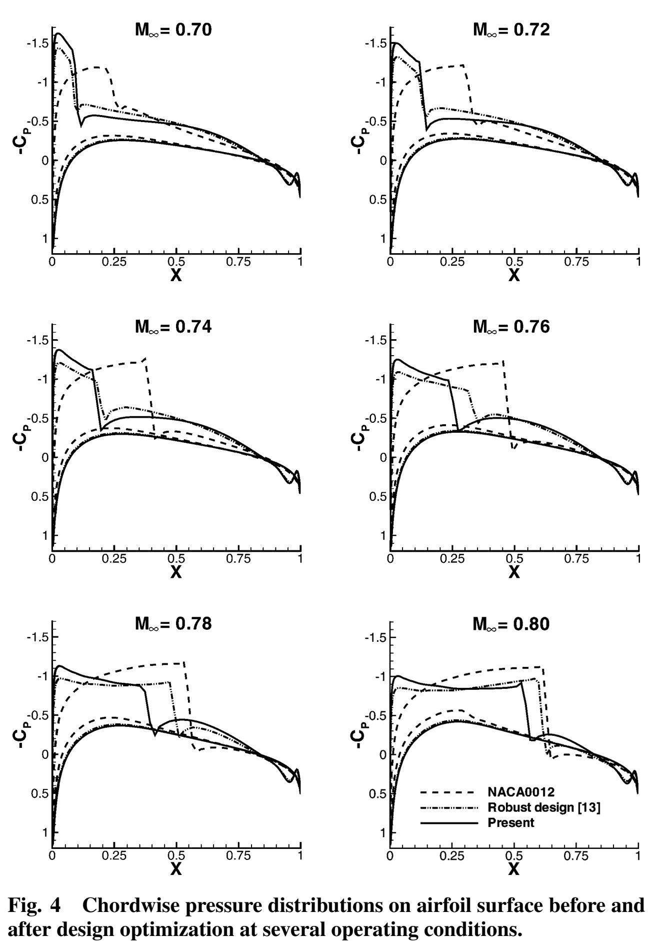

Fig. 4 Chordwise pressure distributions on airfoil surface before and after design optimization at several operating conditions.

Table 1 Mean and standard deviation of the drag coefficient before and after design optimization

Variables	μ	σ
NACA0012	10.387×10^{-3}	6.2686×10^{-3}
Single point design	8.7523×10^{-3}	6.3317×10^{-3}
Multipoint design	6.7260×10^{-3}	2.0299×10^{-3}
Robust design[13]	4.1642×10^{-3}	1.4606×10^{-3}
Present	4.8352×10^{-3}	0.2023×10^{-3}

In Fig. 4, the surface pressure distributions of the designed airfoils are presented at several flight Mach numbers. It shows that, unlike the single-point design optimization, robust design methods considering a range of operating condition do not guarantee the complete elimination of the shock wave. At low freestream Mach numbers, the airfoil designed by the present method shows shock waves slightly stronger than those of the drag minimization method,[13] resulting in slightly higher drag. However, at high freestream Mach numbers, the present optimization method based on design for six sigma predicted weaker shock waves farther upstream, confirming lower wave drag and the effective control of the drag divergence.

The mean and the standard deviation of the drag coefficient obtained from each design method are summarized in Table 1. The airfoil optimized by the present method shows a mean drag significantly lower than that of the original NACA0012 airfoil section and the airfoils obtained from single-point and multipoint design optimizations. Even though the predicted mean drag of the present method was slightly higher than that of the robust design based on drag minimization,[13] the overall variance was significantly reduced by adopting design for six sigma.

Conclusions

In the present study, a robust aerodynamic shape optimization technique has been developed to design airfoils which are free from the drag divergence over a range of transonic flight Mach numbers. For this purpose, the uncertainty of varying operating conditions was modelled by randomly selecting the design points based on a stratified sampling method with equal allocation. The requirement of achieving a consistent performance for uncertain operating conditions was implemented by incorporating statistical parameters such as the mean and the standard deviation of the drag coefficient in the objective function based on the concept of design for six sigma. The present method was applied to the aerodynamic shape optimization of a two-dimensional airfoil, and the results were compared with those of other design optimization methods for validation. It was shown that the present robust aerodynamic shape optimization method based on design for six sigma is a useful tool to enhance the performance of air vehicles operating in transonic flow regimes.

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